

# Using the Package NMF

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This vignette presents the package NMF [?], which implements a framework for Nonnegative Matrix Factorization (NMF) algorithms in R [R Software, 2008]. The objective is to provide an implementation of some standard algorithms, while allowing the user to easily implement new methods that integrate into the package's framework.

To cite the package NMF in publications please use:

Renaud Gaujoux, Cathal Seoighe (2010).

**A flexible R package for nonnegative matrix factorization.**

*BMC Bioinformatics* 2010, **11**:367,

<http://www.biomedcentral.com/1471-2105/11/367>.

The latest stable version of the NMF package can be installed from any CRAN repository mirror, and loaded with the standard calls to `library`.

```
# Install
install.packages('NMF')
# Load
library(NMF)
```

# Contents

<b>1 Overview</b>	<b>3</b>
1.1 Package features	3
1.2 Nonnegative Matrix Factorization	3
1.3 Algorithms	4
1.4 Initialization: seeding methods	4
1.5 How to run NMF algorithms	6
1.6 Performances	7
1.7 How to cite the package NMF	8
<b>2 Use case: Golub dataset</b>	<b>8</b>
2.1 Single run	8
2.1.1 Performing a single run	8
2.1.2 Handling the result	9
2.1.3 Extracting metagene-specific features	11
2.2 Specifying the algorithm	11
2.2.1 Built-in algorithms	11
2.2.2 Custom algorithms	13
2.3 Specifying the seeding method	13
2.3.1 Built-in seeding methods	13
2.3.2 Numerical seed	14
2.3.3 Fixed factorization	14
2.3.4 Custom function	15
2.4 Multiple runs	15
2.4.1 Parallel computing on multi-core machines (non-Windows)	16
2.4.2 High Performance Computing on a cluster	17
2.4.3 Forcing sequential execution	19
2.5 Estimating the factorization rank	19
2.5.1 Overfitting	20
2.6 Visualization methods	21
2.7 Comparing algorithms	24
<b>3 Extending the package</b>	<b>25</b>
3.1 Custom algorithm	26
3.1.1 Using a custom algorithm	26
3.1.2 Using a custom distance measure	27
3.1.3 Defining algorithms for mixed sign data	28
3.1.4 Specifying the NMF model	29
3.2 Custom seeding method	31
<b>4 Advanced usage</b>	<b>32</b>
4.1 Package specific options	32
<b>5 Session Info</b>	<b>32</b>
<b>References</b>	<b>33</b>

# 1 Overview

## 1.1 Package features

This section provides a quick overview of the NMF package’s features. Section 2 provides more details, as well as sample code on how to actually perform common tasks in NMF analysis.

The NMF package provides:

- 7 built-in algorithms;
- 4 built-in seeding methods;
- Single interface to perform all algorithms, and combine them with the seeding methods;
- Provides a common framework to test, compare and develop NMF methods;
- Accept custom algorithms and seeding methods;
- Plotting utility functions to visualize and help in the interpretation of the results;
- Transparent parallel computations;
- Optimized and memory efficient C++ implementations of the standard algorithms;
- Optional layer for bioinformatics using BioConductor [Gentleman *et al.*, 2004];

## 1.2 Nonnegative Matrix Factorization

This section gives a formal definition for Nonnegative Matrix Factorization problems, and defines the notations used throughout the vignette.

Let  $X$  be a  $n \times p$  non-negative matrix, (i.e with  $x_{ij} \geq 0$ , denoted  $X \geq 0$ ), and  $r > 0$  an integer. Non-negative Matrix Factorization (NMF) consists in finding an approximation

$$X \approx WH, \quad (1)$$

where  $W, H$  are  $n \times r$  and  $r \times p$  non-negative matrices, respectively. In practice, the factorization rank  $r$  is often chosen such that  $r \ll \min(n, p)$ . The objective behind this choice is to summarize and split the information contained in  $X$  into  $r$  factors: the columns of  $W$ .

Depending on the application field, these factors are given different names: basis images, metagenes, source signals. In this vignette we equivalently and alternatively use the terms *basis matrix* or *metagenes* to refer to matrix  $W$ , and *mixture coefficient matrix* and *metagene expression profiles* to refer to matrix  $H$ .

The main approach to NMF is to estimate matrices  $W$  and  $H$  as a local minimum:

$$\min_{W, H \geq 0} \underbrace{[D(X, WH) + R(W, H)]}_{=F(W, H)} \quad (2)$$

where

- $D$  is a loss function that measures the quality of the approximation. Common loss functions are based on either the Frobenius distance

$$D : A, B \mapsto \frac{\text{Tr}(AB^t)}{2} = \frac{1}{2} \sum_{ij} (a_{ij} - b_{ij})^2,$$

or the Kullback-Leibler divergence.

$$D : A, B \mapsto KL(A||B) = \sum_{i,j} a_{ij} \log \frac{a_{ij}}{b_{ij}} - a_{ij} + b_{ij}.$$

- $R$  is an optional regularization function, defined to enforce desirable properties on matrices  $W$  and  $H$ , such as smoothness or sparsity [A. Cichocki *et al.*, 2004].

### 1.3 Algorithms

NMF algorithms generally solve problem (2) iteratively, by building a sequence of matrices  $(W_k, H_k)$  that reduces at each step the value of the objective function  $F$ . Beside some variations in the specification of  $F$ , they also differ in the optimization techniques that are used to compute the updates for  $(W_k, H_k)$ .

For reviews on NMF algorithms see [Berry *et al.*, 2006, Chu *et al.*, 2004] and references therein.

The NMF package implements a number of published algorithms, and provides a general framework to implement other ones. Table 1 gives a short description of each one of the built-in algorithms:

The built-in algorithms are listed or retrieved with function `nmfAlgorithm`. A given algorithm is retrieved by its name (a `character` key), that is partially matched against the list of available algorithms:

```
# list all available algorithms
nmfAlgorithm()
[1] "brunet" "lee"    "nsNMF"  "offset" "pe-nmf" "snmf/l"
[7] "snmf/r"

# retrieve a specific algorithm: 'brunet'
nmfAlgorithm('brunet')
<object of class: NMFStrategyIterative >
name:          brunet
objective:      'KL'
NMF model:      NMFstd
<Iterative schema:>
Update : 'nmf.update.brunet'
Stop : 'nmf.stop.consensus'
WrapNMF : ''

# partial match is also fine
identical(nmfAlgorithm('br'), nmfAlgorithm('brunet'))
[1] TRUE
```

### 1.4 Initialization: seeding methods

NMF algorithms need to be initialized with a seed (i.e. a value for  $W_0$  and/or  $H_0$ <sup>1</sup>), from which to start the iteration process. Because there is no global minimization algorithm, and due to the problem's high dimensionality, the choice of the initialization is in fact very important to ensure meaningful results.

The more common seeding method is to use a random starting point, where the entries of  $W$  and/or  $H$  are drawn from a uniform distribution, usually within the same range as the target matrix's entries. This method is very simple to implement. However, a drawback is that to achieve stability one has to perform multiple runs, each with a different starting point. This significantly increases the computation time needed to obtain the desired factorization.

To tackle this problem, some methods have been proposed so as to compute a reasonable starting point from the target matrix itself. Their objective is to produce deterministic algorithms that need to run only once, still giving meaningful results.

For a review on some existing NMF initializations see [Albright *et al.*, 2006] and references therein.

---

<sup>1</sup>Some algorithms only need one matrix factor (either  $W$  or  $H$ ) to be initialized. See for example the SNMF/R(L) algorithm of Kim and Park [Kim and Park, 2007].

Key	Description
<b>brunet</b>	Standard NMF. Based on Kullbach-Leibler divergence, it uses simple multiplicative updates from [Lee and Seung, 2000], enhanced to avoid numerical underflow. $H_{kj} \leftarrow H_{kj} \frac{\left(\sum_l \frac{W_{lk} V_{lj}}{(WH)_{lj}}\right)}{\sum_l W_{lk}} \quad (3)$ $W_{ik} \leftarrow W_{ik} \frac{\sum_l [H_{kl} A_{il} / (WH)_{il}]}{\sum_l H_{kl}} \quad (4)$ <p><b>Reference:</b> [Brunet <i>et al.</i>, 2004]</p>
<b>lee</b>	Standard NMF. Based on euclidean distance, it uses simple multiplicative updates $H_{kj} \leftarrow H_{kj} \frac{(W^T V)_{kj}}{(W^T W H)_{kj}} \quad (5)$ $W_{ik} \leftarrow W_{ik} \frac{(V H^T)_{ik}}{(W H H^T)_{ik}} \quad (6)$ <p><b>Reference:</b> [Lee and Seung, 2000]</p>
<b>nsNMF</b>	Non-smooth NMF. Uses a modified version of Lee and Seung’s multiplicative updates for Kullbach-Leibler divergence to fit a extension of the standard NMF model. It is meant to give sparser results. <p><b>Reference:</b> [Pascual-Montano <i>et al.</i>, 2006]</p>
<b>offset</b>	Uses a modified version of Lee and Seung’s multiplicative updates for euclidean distance, to fit a NMF model that includes an intercept. <p><b>Reference:</b> [Badea L., 2008]</p>
<b>pe-nmf</b>	Pattern-Expression NMF. Uses multiplicative updates to minimize an objective function based on the Euclidean distance and regularized for effective expression of patterns with basis vectors. <p><b>Reference:</b> [Zhang <i>et al.</i>, 2008]</p>
<b>snmf/r, snmf/l</b>	Alternating Least Square (ALS) approach. It is meant to be very fast compared to other approaches. <p><b>Reference:</b> [Kim and Park, 2007]</p>

Table 1: Description of the implemented NMF algorithms. The first column gives the key to use in the call to the `nmf` function.

The `NMF` package implements a number of already published seeding methods, and provides a general framework to implement other ones. Table 2 gives a short description of each one of the built-in seeding methods:

The built-in seeding methods are listed or retrieved with function `nmfSeed`. A given seeding method is retrieved by its name (a `character` key) that is partially matched against the list of available seeding methods:

```
# list all available seeding methods
nmfSeed()
[1] "ica"      "nndsvd"  "none"    "random"
```

```
# retrieve a specific method: 'nndsvd'
nmfSeed('nndsvd')
<object of class: NMFSeed >
name:      nndsvd
method:    <function>

# partial match is also fine
identical(nmfSeed('nn'), nmfSeed('nndsvd'))
[1] TRUE
```

Key	Description
<b>ica</b>	Uses the result of an Independent Component Analysis (ICA) (from the <b>fastICA</b> package). Only the positive part of the result are used to initialize the factors.
<b>nnsvd</b>	Nonnegative Double Singular Value Decomposition. The basic algorithm contains no randomization and is based on two SVD processes, one approximating the data matrix, the other approximating positive sections of the resulting partial SVD factors utilizing an algebraic property of unit rank matrices. It is well suited to initialize NMF algorithms with sparse factors. Simple practical variants of the algorithm allows to generate dense factors. <b>Reference:</b> <a href="#">[Boutsidis and Gallopoulos, 2008]</a>
<b>none</b>	Fix seed. This method allows the user to manually provide initial values for both matrix factors.
<b>random</b>	The entries of each factors are drawn from a uniform distribution over $[0, \max(V)]$ , where $V$ is the target matrix.

Table 2: Description of the implemented seeding methods to initialize NMF algorithms. The first column gives the key to use in the call to the **nmf** function.

## 1.5 How to run NMF algorithms

Method **nmf** provides a single interface to run NMF algorithms. It can directly perform NMF on object of class **matrix** or **data.frame** and **ExpressionSet** – if the **Biobase** package is installed. The interface has four main parameters:

```
nmf(x, rank, method, seed, ...)
```

**x** is the target **matrix**, **data.frame** or **ExpressionSet** <sup>2</sup>

**rank** is the factorization rank, i.e. the number of columns in matrix  $W$ .

**method** is the algorithm used to estimate the factorization. The default algorithm is given by the package specific option '**default.algorithm**', which defaults to '**brunet**' on installation [\[Brunet et al., 2004\]](#).

**seed** is the seeding method used to compute the starting point. The default method is given by the package specific option '**default.seed**', which defaults to '**random**' on initialization (see method **?nmf** for details on its implementation).

See also **?nmf** for details on the interface and extra parameters.

<sup>2</sup>**ExpressionSet** is the base class for handling microarray data in BioConductor, and is defined in the **Biobase** package.

## 1.6 Performances

Since version 0.4, some built-in algorithms are optimized in C++, which results in a significant speed-up and a more efficient memory management, especially on large scale data.

The older R versions of the concerned algorithms are still available, and accessible by adding the prefix '.R#' to the algorithms' access keys (e.g. the key '.R#offset' corresponds to the R implementation of NMF with offset [Badea L., 2008]). Moreover they do not show up in the listing returned by the `nmfAlgorithm` function, unless argument `all=TRUE`:

```
nmfAlgorithm(all=TRUE)
[1] ".R#brunet" ".R#lee" ".R#nsNMF" ".R#offset"
[5] "brunet" "lee" "nsNMF" "offset"
[9] "pe-nmf" "snmf/l" "snmf/r"

# to get all the algorithms that have a secondary R version
nmfAlgorithm(type='R')
      brunet      lee      nsNMF      offset
".R#brunet" ".R#lee" ".R#nsNMF" ".R#offset"
```

Table 3 shows the speed-up achieved by the algorithms that benefit from the optimized code. All algorithms were run once with a factorization rank equal to 3, on the Golub data set which contains a  $5000 \times 38$  gene expression matrix. The same numeric random seed (`seed=123456`) was used for all factorizations. The columns *C* and *R* show the elapsed time (in seconds) achieved by the C++ version and R version respectively. The column *Speed.up* contains the ratio  $R/C$ .

```
# retrieve all the methods that have a secondary R version
meth <- nmfAlgorithm(type='R')
meth <- c(names(meth), meth)
meth

      brunet
"brunet"      "lee"      "nsNMF"      "offset" ".R#brunet"
      lee      nsNMF      offset
".R#lee" ".R#nsNMF" ".R#offset"

# load the Golub data
data(esGolub)
# compute NMF for each method
res <- nmf(esGolub, 3, meth, seed=123456)
# extract only the elapsed time
t <- sapply(res, runtime)[3,]
```

	C	R	Speed.up
brunet	4.08	9.94	2.43
lee	7.88	10.87	1.38
nsNMF	7.60	15.58	2.05
offset	7.05	17.04	2.42

Table 3: Performance speed up achieved by the optimized C++ implementation for some of the NMF algorithms.

## 1.7 How to cite the package NMF

To view all the package's bibtex citations, including all vignette(s) and manual(s):

```
# plain text
citation('NMF')
# or to get the bibtex entries
toBibtex(citation('NMF'))
```

## 2 Use case: Golub dataset

We illustrate the functionalities and the usage of the NMF package on the – now standard – Golub dataset on leukemia. It was used in several papers on NMF [Brunet *et al.*, 2004, Gao and Church, 2005] and is included in the NMF package's data, wrapped into an `ExpressionSet` object. For performance reason we use here only the first 200 genes. Therefore the results shown in the following are not meant to be biologically meaningful, but only illustrative:

```
data(esGolub)
esGolub
ExpressionSet (storageMode: lockedEnvironment)
 assayData: 5000 features, 38 samples
   element names: exprs
 protocolData: none
 phenoData
   sampleNames: ALL_19769_B-cell ALL_23953_B-cell ...
               AML_7 (38 total)
   varLabels: Sample ALL.AML Cell
   varMetadata: labelDescription
 featureData
   featureNames: M12759_at U46006_s_at ... D86976_at
               (5000 total)
   fvarLabels: Description
   fvarMetadata: labelDescription
 experimentData: use 'experimentData(object)'
 Annotation:

esGolub <- esGolub[1:200,]
# remove the unneeded variable 'Sample' from the phenotypic data
esGolub$Sample <- NULL
```

**Note:** To run this example, the `Biobase` package from BioConductor is required.

### 2.1 Single run

#### 2.1.1 Performing a single run

To run the default NMF algorithm on data `esGolub` with a factorization rank of 3, we call:



```
# default NMF algorithm
res <- nmf(esGolub, 3)
```

Here we did not specify either the algorithm or the seeding method, so that the computation is done using the default algorithm and is seeded by the default seeding methods. These defaults are set in the package specific options `'default.algorithm'` and `'default.seed'` respectively.

See also Sections 2.2 and 2.3 for how to explicitly specify the algorithm and/or the seeding method.

### 2.1.2 Handling the result

The result of a single NMF run is an object of class `NMFfit`, that holds both the fitted NMF model and data about the run:

```
res
<Object of class: NMFfit >
# Model:
  <Object of class: NMFstd >
  features: 200
  basis/rank: 3
  samples: 38
# Details:
  algorithm: brunet
  seed: random
  RNG: 403, 624, 452835868, 968717421, 1343013962, -2072317629, 1173789096, ... [ed7ba52c9c2666ca159b1859a]
  distance metric: 'KL'
  residuals: 543535.7
  Iterations: 510
Timing:
  user  system elapsed
 0.360   0.000   0.359
```

The fitted model can be retrieved via method `fit`, which returns an object of class `NMF`:

```
fit(res)
<Object of class: NMFstd >
features: 200
basis/rank: 3
samples: 38
```

The estimated target matrix can be retrieved via the generic method `fitted`, which returns a – generally big – matrix:

```
V.hat <- fitted(res)
dim(V.hat)
[1] 200 38
```

Quality and performance measures about the factorization are computed by method `summary`:

```
summary(res)
      rank sparseness.basis sparseness.coef
3.000000e+00 6.392676e-01 6.217884e-01
  residuals          niter           cpu
5.435357e+05 5.100000e+02 3.600000e-01
   cpu.all          nrun
3.600000e-01 1.000000e+00

# More quality measures are computed, if the target matrix is provided:
summary(res, target=esGolub)
      rank sparseness.basis sparseness.coef
3.000000e+00 6.392676e-01 6.217884e-01
   rss          evar      residuals
1.535504e+09 8.232656e-01 5.435357e+05
   niter          cpu      cpu.all
5.100000e+02 3.600000e-01 3.600000e-01
   nrun
1.000000e+00
```

If there is some prior knowledge of classes present in the data, some other measures about the unsupervised clustering's performance are computed (purity, entropy, ...). Here we use the phenotypic variable `Cell` found in the Golub dataset, that gives the samples' cell-types (it is a factor with levels: T-cell, B-cell or NA):

```
summary(res, class=esGolub$Cell)
      rank sparseness.basis sparseness.coef
3.000000e+00 6.392676e-01 6.217884e-01
   purity          entropy      residuals
8.157895e-01 3.926954e-01 5.435357e+05
   niter          cpu      cpu.all
5.100000e+02 3.600000e-01 3.600000e-01
   nrun
1.000000e+00
```

The basis matrix (i.e. matrix  $W$  or the metagenes) and the mixture coefficient matrix (i.e. matrix  $H$  or the metagene expression profiles) are retrieved using methods `basis` and `coef` respectively:

```
# get matrix W
w <- basis(res)
dim(w)
[1] 200 3

# get matrix H
h <- coef(res)
dim(h)
[1] 3 38
```

If one wants to keep only part of the factorization, one can directly subset on the NMF object on features and samples (separately or simultaneously). The result is a NMF object composed of the selected rows and/or columns:

```
# keep only the first 10 features
res.subset <- res[1:10,]
class(res.subset)
[1] "NMFfit"
attr(,"package")
[1] "NMF"

dim(res.subset)
[1] 10 38 3

# keep only the first 10 samples
dim(res[,1:10])
[1] 200 10 3

# subset both features and samples:
dim(res[1:20,1:10])
[1] 20 10 3
```

### 2.1.3 Extracting metagene-specific features

In general NMF matrix factors are sparse, so that the metagenes can usually be characterized by a relatively small set of genes. Those are determined based on their relative contribution to each metagene.

Kim and Park [Kim and Park, 2007] defined a procedure to extract the relevant genes for each metagene, based on a gene scoring schema.

The NMF package implements this procedure in methods `featureScore` and `extractFeature`:

```
# only compute the scores
s <- featureScore(res)
summary(s)
      Min.   1st Qu.   Median     Mean   3rd Qu.    Max.
0.0001208 0.0162700 0.0548900 0.1185000 0.1210000 1.0000000

# compute the scores and characterize each metagene
s <- extractFeatures(res)
str(s)
List of 3
 $ 1: int [1:8] 2 39 74 91 103 167 174 190
 $ 2: int [1:13] 1 8 25 26 41 42 59 64 69 94 ...
 $ 3: int [1:5] 43 120 128 129 130
 - attr(*, "threshold")= num 0.251
```

## 2.2 Specifying the algorithm

### 2.2.1 Built-in algorithms

The NMF package provides a number of built-in algorithms, that are listed or retrieved by function `nmfAlgorithm`. Each algorithm is identified by a unique name. The following algorithms are

currently implemented (cf. Table 1 for more details):

```
nmfAlgorithm()  
[1] "brunet" "lee"      "nsNMF"  "offset" "pe-nmf" "snmf/l"  
[7] "snmf/r"
```

The algorithm used to compute the NMF is specified in the third argument (`method`). For example, to use the NMF algorithm from Lee and Seung [Lee and Seung, 2000] based on the Frobenius euclidean norm, one make the following call:

```
# using Lee and Seung's algorithm  
res <- nmf(esGolub, 3, 'lee')  
algorithm(res)  
[1] "lee"
```

To use the Nonsmooth NMF algorithm from [Pascual-Montano *et al.*, 2006]:

```
# using the Nonsmooth NMF algorithm with parameter theta=0.7  
res <- nmf(esGolub, 3, 'ns', theta=0.7)  
algorithm(res)  
[1] "nsNMF"  
  
fit(res)  
<Object of class: NMFns >  
features: 200  
basis/rank: 3  
samples: 38  
theta: 0.7
```

Or to use the PE-NMF algorithm from [Zhang *et al.*, 2008]:

```
# using the PE-NMF algorithm with parameters alpha=0.01, beta=1  
res <- nmf(esGolub, 3, 'pe', alpha=0.01, beta=1)  
res  
<Object of class: NMFfit >  
# Model:  
  <Object of class: NMFstd >  
    features: 200  
    basis/rank: 3  
    samples: 38  
# Details:  
  algorithm: pe-nmf  
  seed: random  
  RNG: 403, 270, 726613627, -497877895, 1428077817, 903502352, -764738250, ... [9f02c5e72ae1205c96d271bdf]  
  distance metric: <function>  
  residuals: 67.35798
```

```

parameters:
$alpha
[1] 0.01

$beta
[1] 1

Iterations: 2000
Timing:
  user  system elapsed
 1.450   0.000   1.441

```

### 2.2.2 Custom algorithms

The NMF package provides the user the possibility to define his own algorithms, and benefit from all the functionalities available in the NMF framework. There are only few constraints on the way the custom algorithm must be defined. See the details in Section [3.1.1](#).

## 2.3 Specifying the seeding method

The seeding method used to compute the starting point for the chosen algorithm can be set via argument `seed`. Note that if the seeding method is deterministic there is no need to perform multiple run anymore.

### 2.3.1 Built-in seeding methods

Similarly to the algorithms, the `nmfSeed` function can be used to list or retrieve the built-in seeding methods. The following seeding methods are currently implemented:

```

nmfSeed()
[1] "ica"      "nndsvd" "none"    "random"

```

To use a specific method to seed the computation of a factorization, one simply passes its name to `nmf`:

```

res <- nmf(esGolub, 3, seed='nndsvd')
res
<Object of class: NMFFit >
# Model:
<Object of class: NMFstd >
features: 200
basis/rank: 3
samples: 38
# Details:
algorithm: brunet
seed: nndsvd
RNG: 403, 360, -1643107813, 805722583, -1295441775, -1201449000, 1434032699, ... [db0a30b7ce8968b8eba699]
distance metric: 'KL'
residuals: 547143.5

```

```
Iterations: 1090
Timing:
  user  system elapsed
0.780   0.000   0.786
```

### 2.3.2 Numerical seed

Another possibility, useful when comparing methods or reproducing results, is to set the random number generator (RNG) by passing a numerical value in argument `seed`. This value is used to set the state of the RNG, and the initialization is performed by the built-in seeding method `'random'`. When the function `nmf` exits, the value of the random seed (`.Random.seed`) is restored to its original state – as before the call.

In the case of a single run (i.e. with `nrun=1`), the default is to use the current RNG, set with the R core function `set.seed`. In the case of multiple runs, the computations use `RNGstream`, as provided by the core RNG “L’Ecuyer-CMRG” [?], which generates multiple independent random streams (one per run). This ensures the complete reproducibility of any given set of runs, even when their computation is performed in parallel. Since `RNGstream` requires a 6-length numeric seed, a random one is generated if only a single numeric value is passed to `seed`. Moreover, single runs can also use `RNGstream` by passing a 6-length seed.

```
# single run and single numeric seed
res <- nmf(esGolub, 3, seed=123456)
RNGinfo(res)
  RNG kind:  Mersenne-Twister / Inversion
  RNG state: 403, 624, 452835868, 968717421, 1343013962, -2072317629, 1173789096, ... [ed7ba52c9c2666ca159b]

# multiple runs and single numeric seed
res <- nmf(esGolub, 3, seed=123456, nrun=2)
RNGinfo(res)
  RNG kind:  L'Ecuyer-CMRG / Inversion
  RNG state: 407, -1104003193, 452835868, 968717421, 1343013962, -2072317629, 1173789096

# single run with a 6-length seed
res <- nmf(esGolub, 3, seed=rep(123456, 6))
RNGinfo(res)
  RNG kind:  L'Ecuyer-CMRG / Inversion
  RNG state: 407, 123456, 123456, 123456, 123456, 123456
```

**NB:** To show the RNG changes happening during the computation use `.options='v4'` to turn on verbosity at level 4. In versions prior 0.6, one could specify option `restore.seed=FALSE` or `'-r'`, this option is now deprecated.

### 2.3.3 Fixed factorization

Yet another option is to completely specify the initial factorization, by passing values for matrices  $W$  and  $H$ :

```
# initialize a "constant" factorization based on the target dimension
init <- nmfModel(3, esGolub, W=0.5, H=0.3)
head(basis(init))

      [,1] [,2] [,3]
M12759_at    0.5  0.5  0.5
U46006_s_at  0.5  0.5  0.5
X70083_at    0.5  0.5  0.5
X03100_cds2_at 0.5  0.5  0.5
L32976_at    0.5  0.5  0.5
M19878_s_at  0.5  0.5  0.5

# fit using this NMF object
res <- nmf(esGolub, 3, seed=init)
```

### 2.3.4 Custom function

The NMF package provides the user the possibility to define his own seeding method, and benefit from all the functionalities available in the NMF framework. There are only few constraints on the way the custom seeding method must be defined. See the details in [Section 3.2](#).

## 2.4 Multiple runs

When the seeding method is stochastic, multiple runs are usually required to achieve stability or a resonable result. This can be done by setting argument `nrun` to the desired value. For performance reason we use `nrun=5` here, but a typical choice would lies between 100 and 200:

```
res.multirun <- nmf(esGolub, 3, nrun=5)
res.multirun
<Object of class: NMFfitX1 >
  Method: brunet
  Runs: 5
  RNG:
    407, 2060366693, -1979266846, -1467721989, 23029248, 1469230593, 1095525326
  Total timing:
    user system elapsed
    2.320  0.190  2.397
```

By default, the returned object only contains the best fit over all the runs. That is the factorization that achieved the lowest approximation error (i.e. the lowest objective value). Even during the computation, only the current best factorization is kept in memory. This limits the memory requirement for performing multiple runs, which in turn allows to perform more runs.

The object `res.multirun` is of class `NMFfitX1` that extends class `NMFfit`, the class returned by single NMF runs. It can therefore be handled as the result of a single run and benefit from all the methods defined for single run results.

If one is interested in keeping the results from all the runs, one can set the option `keep.all=TRUE`:

```
# explicitly setting the option keep.all to TRUE
res <- nmf(esGolub, 3, nrun=5, .options=list(keep.all=TRUE))
res
```

```

<Object of class: NMffitXn >
Method: brunet
Runs: 5
RNG:
407, 1997333926, -492612625, 469522788, -1828607723, 1996453586, -1167273941
Total timing:
  user system elapsed
2.340  0.180  2.033
Sequential timing:
  user system elapsed
2.340  0.040  2.424

```

```

# or using letter code 'k' in argument .options
nmf(esGolub, 3, nrun=5, .options='k')

```

In this case, the result is an object of class `NMffitXn` that also inherits from class `list`.

Note that keeping all the results may be memory consuming. For example, a 3-rank NMF fit<sup>3</sup> for the Golub gene expression matrix ( $5000 \times 38$ ) takes about 23Kb<sup>4</sup>.

#### 2.4.1 Parallel computing on multi-core machines (non-Windows)

To speed-up the analysis whenever possible, the `NMF` package implements transparent parallel computations when run on multi-core machines. It uses the `foreach` framework developed by Revolution Computing [[foreach, 2009](#)], together with the related `doMC` parallel backend [[doMC, 2009](#)] – based on the `multicore` package – to make use of all the CPUs available on the system. Each core will simultaneously perform part of the runs. Therefore, the required memory increases linearly with the number of cores used. When only the best run is of interest, the memory usage is optimized by using shared memory and mutex objects from the packages `bigmemory` and `synchronicity`, to only keep the current best factorization.

**IMPORTANT NOTE:** because it uses the `multicore` package, parallel computation over multi-cores is available only for Unix and Mac machines. The parallel computation is based on the `doMC` and `multicore` packages, so the same care should be taken as stated in the vignette of `doMC`:

*... it usually isn't safe to run doMC and multicore from a GUI environment. In particular, it is not safe to use doMC from R.app on Mac OS X. Instead, you should use doMC from a terminal session, starting R from the command line.*

Therefore, the `nmf` function does not allow to run multicore computation from the MacOS X GUI.

The default parallel backend used by the `nmf` function is defined by the package specific option `'parallel.backend'`, which defaults to `'mc'` – for `doMC`. The backend can also be set on runtime via argument `'pbackend'`.

There are two other runtime options, `parallel` and `parallel.required`, that can be passed via argument `.options`, to control the behaviour of the parallel computation (see below).

A call for multiple runs will be computed in parallel if one of the following condition is satisfied:

<sup>3</sup>i.e. the result of a single NMF run with rank equal 3.

<sup>4</sup>This size might change depending on the architecture (32 or 64 bits)



- call with option 'P' or `parallel.required` set to TRUE (note the upper case in 'P'). In this case, if for any reason the computation cannot be run in parallel (packages requirements, OS, ...), then an error is thrown. Use this mode to force the parallel execution.
- call with option 'p' or `parallel` set to TRUE. In this case if something prevents a parallel computation, the factorizations will be done sequentially.
- a valid parallel backend is specified in argument `.pbackend`. For the moment it can either be the string 'mc' or a single numeric value specifying the number of core to use. Unless option 'P' is specified, it will run using option 'p' (i.e. try-parallel mode).

**NB:** The number of processors to use can also be specified in the runtime options as e.g. `.options='p4'` or `.options='P4'` – to ask or request 4 CPUs.

## Examples

The following examples are run with `.options='v2'` which turn on verbosity at level 2. However *Sweave* do not show all the messages. The user is therefore encouraged to run these commands on his machine to see the internal differences of each call.

```
# the default call will try to run in parallel using all the cores
# => will be in parallel if all the requirements are satisfied
nmf(esGolub, 3, nrun=5, .opt='v')
Runs: 2 1* 4 3 5 ... DONE
System time:
  user  system elapsed
 3.030   0.200   2.508
<Object of class: NMffitX1 >
Method: brunet
Runs: 5
RNG:
407, 479565376, 313168193, 758619662, -1499576521, -1114385908, 588631965
Total timing:
  user  system elapsed
 3.030   0.200   2.508
```

```
# request a certain number of cores to use => no error if not possible
nmf(esGolub, 3, nrun=5, .opt='vp8')
# force parallel computation: use option 'P'
nmf(esGolub, 3, nrun=5, .opt='vP')
# require an improbable number of cores => error
nmf(esGolub, 3, nrun=5, .opt='vP200')
```

### 2.4.2 High Performance Computing on a cluster

To achieve further speed-up, the computation can be run on an HPC cluster. In our tests we used the `doMPI` package to perform 100 factorizations using hybrid parallel computation on 4 quadri-core machines – making use of all the cores computation on each machine.

The scripts used to launch and run the factorizations can be found in file *mpi.R* in the package's *examples* directory:

```
file.show(system.file('examples/mpi.R', package='NMF'))
# and
file.show(system.file('examples/mpi_run.sh', package='NMF'))
```

The script file *mpi.R* contains some extra code to log and trace the computation. Reducing it to the essential gives the following piece of code:

```
## 0. Create and register an MPI cluster
library(doMPI)
cl <- startMPIcluster()
registerDoMPI(cl)
library(NMF)
## 1. Schedule the runs accross the workers
nrun <- 100;
nworker <- getDoParWorkers();
ntasks <- rep(round(nrun/nworker), nworker)
# allocate remainder runs
if( (remain <- nrun %% nworker) > 0 )
  ntasks[1:remain] <- ntasks[1:remain] + 1
## 2. Send the jobs to the workers using a foreach loop
t <- system.time({
  res <- foreach(i=1:getDoParWorkers(), n=ntasks,
    .packages = c('NMF', 'doMC', 'Biobase')) %dopar% {

    # each worker run its factorizations in parallel
    #Note: only the best result is kept
    data(esGolub)
    nmf(esGolub, 3, 'brunet', nrun=n, .opt='p')

  }
})
## 3. reduce the result and save it in a file
res <- NMF:::join(res, runtime.all=t)
save(res, file='result.RData')
## 4. Shutdown the cluster and quit MPI
closeCluster(cl)
mpi.quit()
```

Passing the following shell script to *qsub* should launch the execution on a Sun Grid Engine HPC cluster, with OpenMPI. Some adaptation might be necessary for other queueing systems.

```
#!/bin/bash
#$ -cwd
```

```

## -q opteron.q
## -pe mpich 5
echo "Got $NSLOTS slots. $TMP/machines"

orterun -v -n $NSLOTS -hostfile $TMP/machines R --slave -f mpi.R

```

### 2.4.3 Forcing sequential execution

When running on a single core machine, NMF package has no other option than performing the multiple runs sequentially, one after another. This is done via the `sapply` function.

On multi-core machine, one usually wants to perform the runs in parallel, as it speeds up the computation (cf. Section 2.4.1). However in some situation (e.g. while debugging), it might be useful to force the sequential execution of the runs. This can be done via the option `'-p'` or `'p1'` by setting the parallel backend to `NULL`, `'seq'` or `''`:

```

# force sequential computation by sapply: use option '-p' or .pbackend=''
nmf(esGolub, 3, nrun=5, .opt='v-p')
nmf(esGolub, 3, nrun=5, .opt='v', .pbackend='')
# or use the SEQ backend of foreach: .pbackend=NULL or 'seq'
nmf(esGolub, 3, nrun=5, .opt='v', .pbackend=NULL)
nmf(esGolub, 3, nrun=5, .opt='v', .pbackend='seq')
nmf(esGolub, 3, nrun=5, .opt='vp1')

```

## 2.5 Estimating the factorization rank

A critical parameter in NMF is the factorization rank  $r$ . It defines the number of metagenes used to approximate the target matrix. Given a NMF method and the target matrix, a common way of deciding on  $r$  is to try different values, compute some quality measure of the results, and choose the best value according to this quality criteria.

Several approaches have then been proposed to choose the optimal value of  $r$ . For example, [Brunet *et al.*, 2004] proposed to take the first value of  $r$  for which the cophenetic coefficient starts decreasing, [Hutchins *et al.*, 2008] suggested to choose the first value where the RSS curve presents an inflection point, and [Frigyesi and Höglund, 2008] considered the smallest value at which the decrease in the RSS is lower than the decrease of the RSS obtained from random data.

The NMF package provides functions to help implement such procedures and plot the relevant quality measures. Note that this can be a lengthy computation, depending on the data size. Whereas the standard NMF procedure usually involves several hundreds of random initialization, performing 30-50 runs is considered sufficient to get a robust estimate of the factorization rank [Brunet *et al.*, 2004, Hutchins *et al.*, 2008]. For performance reason, we perform here only 10 runs for each value of the rank.

```

# perform 10 runs for each value of r in range 2:6
estim.r <- nmf(esGolub, 2:6, nrun=10, seed=123456)

```

The result is a S3 object of class `NMF.rank`, that contains a `data.frame` with the quality measures in column, and the values of  $r$  in row. It also contains a list of the consensus matrix for each value of  $r$ .

```
plot(estim.r)
```

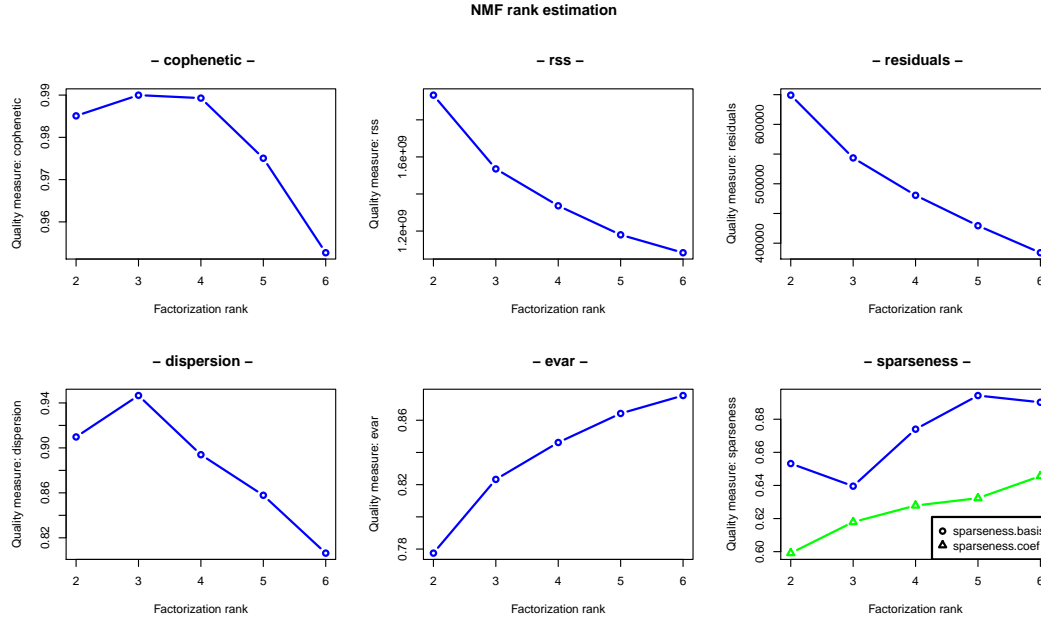


Figure 1: Estimation of the rank: Quality measures computed from 10 runs for each value of  $r$ .

All the measures can be plotted at once with the method `plot` (Figure 1), and the function `consensusmap` generates heatmaps of the consensus matrix for each value of the rank. In the context of class discovery, it is useful to see if the clusters obtained correspond to known classes. This is why in the particular case of the Golub dataset, we added annotation tracks for the two covariates available ('Cell' and 'ALL.AML'). Since we removed the variable 'Sample' in the preliminaries, these are the only variables in the phenotypic `data.frame` embedded within the `ExpressionSet` object, and we can simply pass the whole object to argument `annCol` (Figure 2). One can see that at rank 2, the clusters correspond to the ALL and AML samples respectively, while rank 3 separates AML from ALL/T-cell and ALL/B-cell<sup>5</sup>.

### 2.5.1 Overfitting

Even on random data, increasing the factorization rank would lead to decreasing residuals, as more variables are available to better fit the data. In other words, there is potentially an overfitting problem.

In this context, the approach from [Frigyesi and Höglund, 2008] may be useful to prevent or detect overfitting as it takes into account the results for unstructured data. However it requires to compute the quality measure(s) for the random data. The NMF package provides a function that shuffles the original data, by permuting the rows of each column, using each time a different permutation. The rank estimation procedure can then be applied to the randomized data, and the “random” measures added to the plot for comparison (Figure 3).

<sup>5</sup>Remember that the plots shown in Figure 2 come from only 10 runs, using the 200 first genes in the dataset, which explains the somewhat not so clean clusters. The results are in fact much cleaner when using the full dataset (Figure 6).

```
consensusmap(estim.r, annCol=esGolub)
```

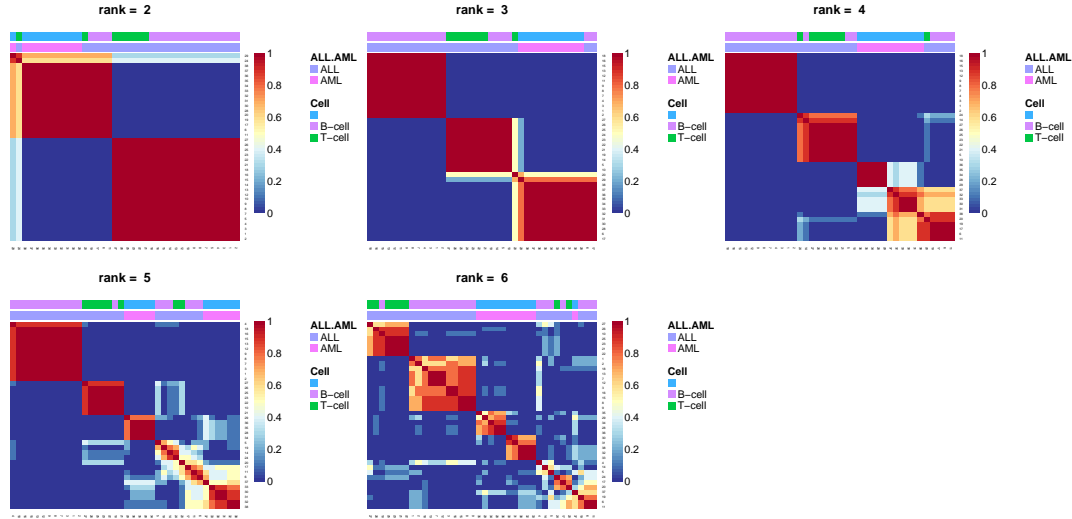


Figure 2: Estimation of the rank: Consensus matrices computed from 10 runs for each value of  $r$ .

## 2.6 Visualization methods

### Error track

If the NMF computation is performed with error tracking enabled – using argument `.options` – the trajectory of the objective value is computed during the fit. This computation is not enabled by default as it induces some overhead.

```
# run nmf with .option='t'
res <- nmf(esGolub, 3, .options='t')
# or with .options=list(track=TRUE)
```

The trajectory can be plot with the method `plot` (Figure 4):

```
plot(res)
```

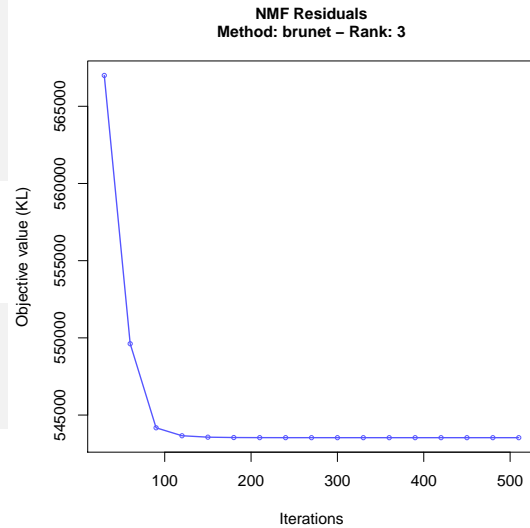


Figure 4: Error track for a single NMF run

```
# shuffle original data
V.random <- randomize(esGolub)
# estimate quality measures from the shuffled data (use default NMF algorithm)
estim.r.random <- nmfEstimateRank(V.random, range=2:6, nrun=10, seed=123456)
```

```
# plot measures on same graph
plot(estim.r, ref=estim.r.random)
```

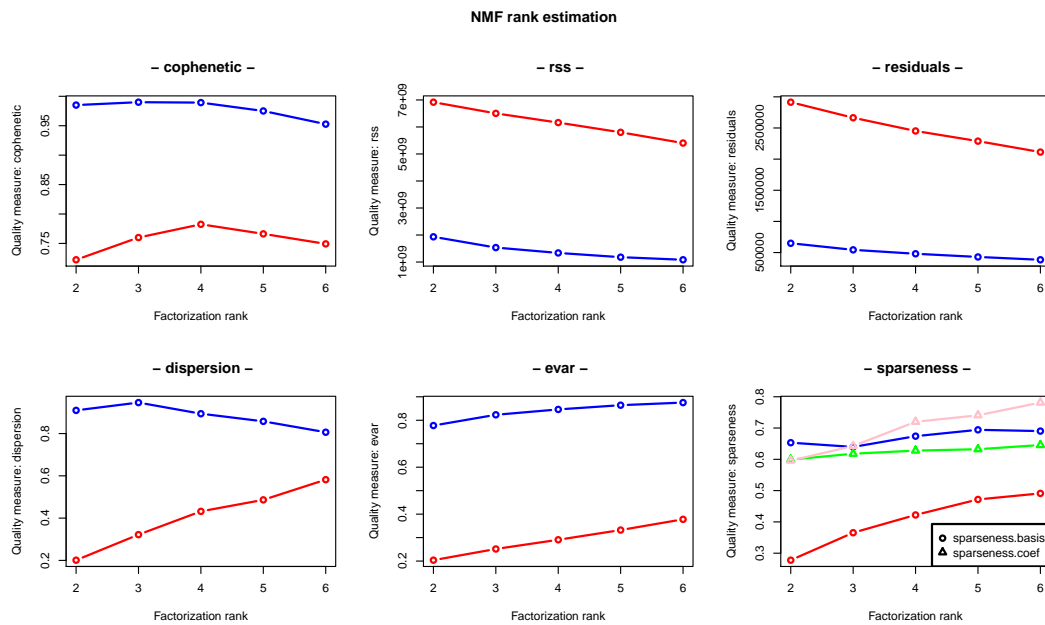


Figure 3: Estimation of the rank: Comparison of the quality measures with those obtained from randomized data. The curves for the actual data are in blue and green, those for the randomized data are in red and pink. The estimation is based on Brunet's algorithm.

## Heatmaps

The methods `basismap`, `coefmap` and `consensusmap` provide an easy way to visualize respectively the resulting basis matrix (i.e. metagenes), mixture coefficient matrix (i.e. metaprofiles) and the consensus matrix, in the case of multiple runs. It produces pre-configured heatmaps based on the function `aheatmap`. The default heatmaps produced by these functions are shown in Figures 5 and 6. They can be customized in many different ways (colours, annotations, labels). See the dedicated vignette “*NMF: generating heatmaps*” or the help pages `?coefmap` and `?aheatmap` for more information.

The rows of the basis matrix often carry the high dimensionality of the data: genes, loci, pixels, features, etc... The function `basismap` extends the use of argument `subsetRow` (from `aheatmap`) to the specification of a feature selection method. In Figure 5 we simply used `subsetRow=TRUE`, which subsets the rows using the method described in [Kim and Park, 2007], to only keep the basis-specific features (e.g. the metagene-specific genes). We refer to the relevant help pages `?basismap` and `?aheatmap` for more details about other possible values for this argument.

```
layout(cbind(1,2))
# basis components
basismap(res, subsetRow=TRUE)
# mixture coefficients
coefmap(res)
```

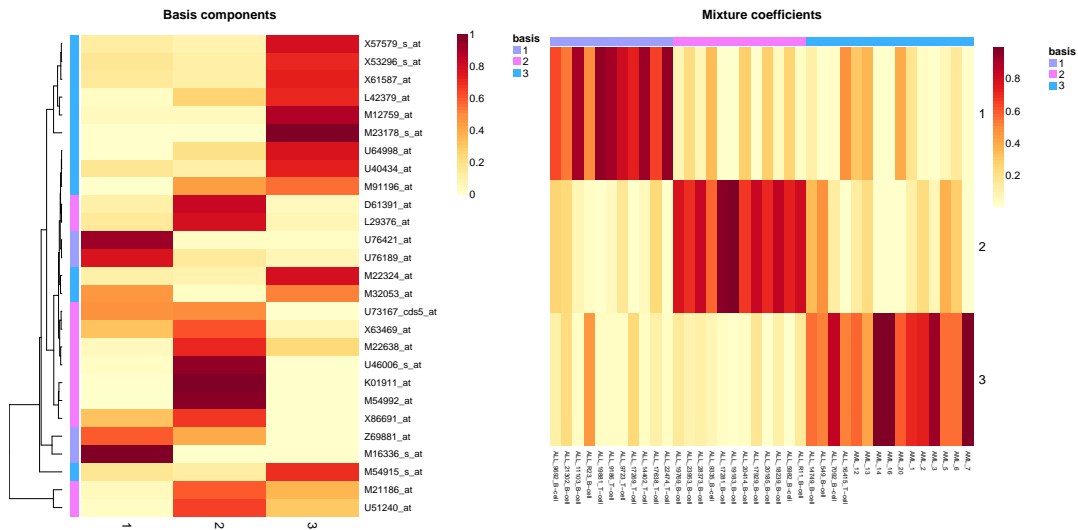


Figure 5: Heatmap of the basis and the mixture coefficient matrices. The rows of the basis matrix were selected using the default feature selection method – described in [Kim and Park, 2007].

**NB:** Note how the function `aheatmap`, the underlying heatmap engine provided with the package `NMF`, makes it is possible to combine several heatmaps on the same plot, using the standard layout call `par(mfrow=...)`. It is also directly compatible with both `layout(...)` and `viewports` from `grid` graphics. The plotting context is automatically detected behind the scene, and a correct behaviour is achieved thanks to the package `gridBase`.

In the case of multiple runs the function `consensusmap` plots the consensus matrix, i.e. the average connectivity matrix across the runs (see results in Figure 6 for a consensus matrix obtained

with 100 runs of Brunet's algorithm on the complete Golub dataset):

```
# The cell type is used to label rows and columns
consensusmap(res.multirun, annCol=esGolub, tracks=NA)
```

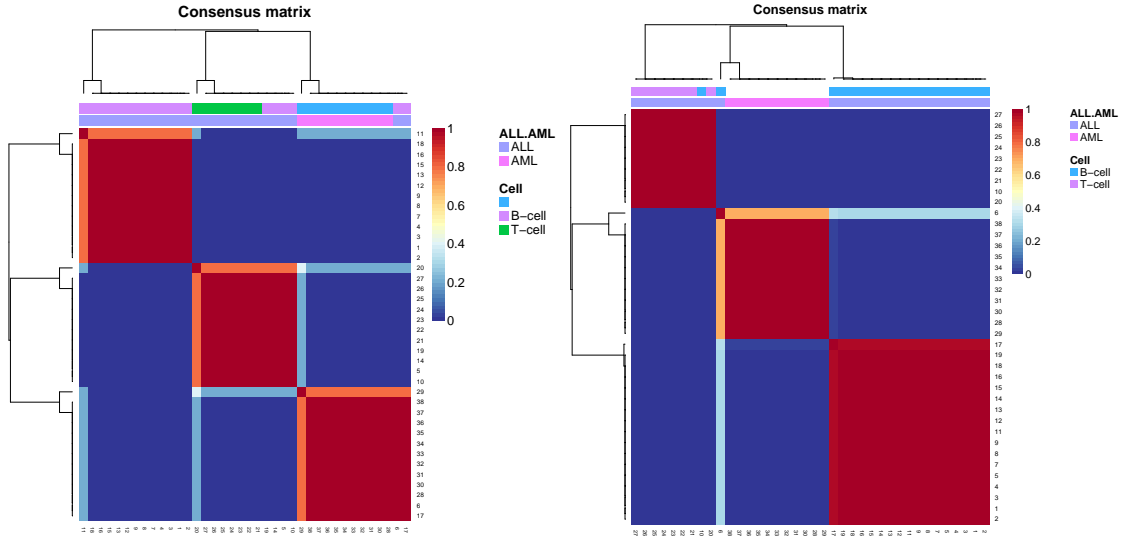


Figure 6: Heatmap of consensus matrices from 10 runs on the limited dataset (left) and from 100 runs on the complete Golub dataset (right).

## 2.7 Comparing algorithms

To compare the results from different algorithms, one can pass a list of methods in argument `method`. To enable a fair comparison, a deterministic seeding method should also be used. Here we fix the random seed to 123456.

```
# fit a model for several different methods
res.multi.method <- nmf(esGolub, 3, list('brunet', 'lee', 'ns'), seed=123456, .options='t')
```

Passing the result to method `compare` produces a `data.frame` that contains summary measures for each method. Again, prior knowledge of classes may be used to compute clustering quality measures:

```
compare(res.multi.method)
  method seed rng metric rank sparseness.basis
brunet brunet random 1 KL 3 0.6392676
lee lee random 1 euclidean 3 0.7268875
nsNMF nsNMF random 1 KL 3 0.6777185
sparseness.coef residuals niter cpu cpu.all nrun
```



brunet	0.6217884	543535.7	510	0.41	0.41	1
lee	0.4465608	673513120.5	1780	1.72	1.72	1
nsNMF	0.7350386	585106.4	970	1.21	1.21	1

	method	seed	rng	metric	rank	sparseness.basis
brunet	brunet	random	1	KL	3	0.6392676
lee	lee	random	1	euclidean	3	0.7268875
nsNMF	nsNMF	random	1	KL	3	0.6777185
	sparseness.coef	purity	entropy	residuals		
brunet	0.6217884	0.8157895	0.3926954	543535.7		
lee	0.4465608	0.5789474	0.7231282	673513120.5		
nsNMF	0.7350386	0.7894737	0.4691212	585106.4		
	niter	cpu	cpu.all	nrun		
brunet	510	0.41	0.41	1		
lee	1780	1.72	1.72	1		
nsNMF	970	1.21	1.21	1		

Because the computation was performed with error tracking enabled, an error plot can be produced by method `plot` (Figure 7). Each track is normalized so that its first value equals one, and stops at the iteration where the method's convergence criterion was fulfilled.

```
plot(res.multi.method)
```

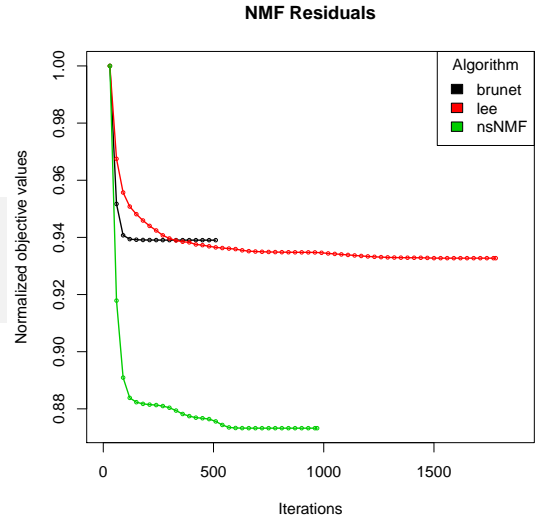


Figure 7: Error tracks comparing methods 'brunet', 'lee', 'nsNMF'

### 3 Extending the package

We developed the `NMF` package with the objective to facilitate the integration of new NMF methods, trying to impose only few requirements on their implementations. All the built-in algorithms and seeding methods are implemented as strategies that are called from within the main interface method `nmf`.

The user can define new strategies and those are handled in exactly the same way as the built-in ones, benefiting from the same utility functions to interpret the results and assess their performance.

## 3.1 Custom algorithm

### 3.1.1 Using a custom algorithm

To define a strategy, the user needs to provide a function that implements the complete algorithm. It must be of the form:

```
my.algorithm <- function(x, seed, param.1, param.2){  
  # do something with starting point  
  # ...  
  
  # return updated starting point  
  return(seed)  
}
```

Where:

**target** is a matrix;

**start** is an object that inherits from class NMF. This S4 class is used to handle NMF models (matrices W and H, objective function, etc...);

**param.1**, **param.2** are extra parameters specific to the algorithms;

The function must return an object that inherits from class NMF.  
For example:

```
my.algorithm <- function(x, seed, scale.factor=1){  
  # do something with starting point  
  # ...  
  # for example:  
  # 1. compute principal components  
  pca <- prcomp(t(x), retx=TRUE)  
  
  # 2. use the absolute values of the first PCs for the metagenes  
  # Note: the factorization rank is stored in object 'start'  
  factorization.rank <- nbasis(seed)  
  metagenes(fit(seed)) <- abs(pca$rotation[,1:factorization.rank])  
  # use the rotated matrix to get the mixture coefficient  
  # use a scaling factor (just to illustrate the use of extra parameters)  
  metaprofiles(fit(seed)) <- t(abs(pca$x[,1:factorization.rank])) / scale.factor  
  
  # return updated data  
  return(seed)  
}
```

To use the new method within the package framework, one pass **my.algorithm** to main interface **nmf** via argument **method**. Here we apply the algorithm to some matrix V randomly generated:

```
n <- 50; r <- 3; p <- 20
V <- syntheticNMF(n, r, p, noise=TRUE)
```

```
nmf(V, 3, my.algorithm, scale.factor=10)
<Object of class: NMFfit >
# Model:
  <Object of class: NMFstd >
  features: 50
  basis/rank: 3
  samples: 20
# Details:
  algorithm: NMF.algo.38980dc4
  seed: random
  RNG: 403, 173, 1191562961, -1256572122, 21108118, 176783409, -1574879598, ... [0d87fde5bf092083bd059bcc3
  distance metric: 'euclidean'
  residuals: 634.5939
  parameters:
  $scale.factor
  [1] 10

Timing:
  user  system elapsed
  0.000   0.000   0.021
```

### 3.1.2 Using a custom distance measure

The default distance measure is based on the euclidean distance. If the algorithm is based on another distance measure, this one can be specified in argument **objective**, either as a **character** string corresponding to a built-in objective function, or a custom **function** definition:

```
# based on Kullbach-Leibler divergence
nmf(V, 3, my.algorithm, scale.factor=10, objective='KL')
<Object of class: NMFfit >
# Model:
  <Object of class: NMFstd >
  features: 50
  basis/rank: 3
  samples: 20
# Details:
  algorithm: NMF.algo.5f1c7268
  seed: random
  RNG: 403, 383, 1191562961, -1256572122, 21108118, 176783409, -1574879598, ... [2f2572f5c6b643d37859d9707
  distance metric: 'KL'
  residuals: 1626.166
  parameters:
  $scale.factor
  [1] 10

Timing:
```

```

      user  system elapsed
0.000    0.000    0.003

<Object of class: NMFfit >
# Model:
  <Object of class: NMFstd >
  features: 50
  basis/rank: 3
  samples: 20
# Details:
  algorithm: NMF.algo.4cbdd64c
  seed: random
  RNG: 403, 593, 1191562961, -1256572122, 21108118, 176783409, -1574879598, ... [7ed64fa772a4b1e68cebc0bf
  distance metric: <function>
  residuals: 9.408703
  parameters:
  $scale.factor
  [1] 10

Timing:
      user  system elapsed
0.010    0.000    0.003

```

### 3.1.3 Defining algorithms for mixed sign data

All the algorithms implemented in the NMF package assume that the input data is nonnegative. However, some methods exist in the literature that work with relaxed constraints, where the input data and one of the matrix factors ( $W$  or  $H$ ) are allowed to have negative entries (eg. semi-NMF [Chris Ding *et al.*, 2008, Le Roux *et al.*, 2008]). Strictly speaking these methods do not fall into the NMF category, but still solve constrained matrix factorization problems, and could be considered as NMF methods when applied to non-negative data. Moreover, we received user requests to enable the development of semi-NMF type methods within the package’s framework. Therefore, we designed the NMF package so that such algorithms – that handle negative data – can be integrated. This section documents how to do it.

By default, as a safe-guard, the sign of the input data is checked before running any method, so that the `nmf` function throws an error if applied to data that contain negative entries <sup>6</sup>. To extend the capabilities of the NMF package in handling negative data, and plug mixed sign NMF methods into the framework, the user needs to specify the argument `mixed=TRUE` in the call to the `nmf` function. This will skip the sign check of the input data and let the custom algorithm perform the factorization.

As an example, we reuse the previously defined custom algorithm<sup>7</sup>:

<sup>6</sup>Note that on the other side, the sign of the factors returned by the algorithms is never checked, so that one can always return factors with negative entries.

<sup>7</sup>As it is defined here, the custom algorithm still returns nonnegative factors, which would not be desirable in a real example, as one would not be able to closely fit the negative entries.

```

# put some negative input data
V.neg <- V; V.neg[1,] <- -1;
# this generates an error
err <- try( nmf(V.neg, 3, my.algorithm, scale.factor=10) )
err

[1] "Error in .local(x, rank, method, ...) : \n  Input matrix x contains some negative entries.\n"
attr("class")
[1] "try-error"
attr("condition")
<simpleError in .local(x, rank, method, ...): Input matrix x contains some negative entries.>

# this runs my.algorithm without error
nmf(V.neg, 3, my.algorithm, mixed=TRUE, scale.factor=10)
<Object of class: NMFfit >
# Model:
  <Object of class: NMFstd >
  features: 50
  basis/rank: 3
  samples: 20
# Details:
  algorithm: NMF.algo.15166fc5
  seed: random
  RNG: 403, 179, -963168695, -1903204175, 1248152975, 1172799541, -1763377217, ... [850b670d66e22c6d00f5d7
  distance metric: 'euclidean'
  residuals: 640.9436
  parameters:
  $scale.factor
  [1] 10

Timing:
      user  system elapsed
0.000   0.000   0.003

```

### 3.1.4 Specifying the NMF model

If not specified in the call, the NMF model that is used is the standard one, as defined in Equation (1). However, some NMF algorithms have different underlying models, such as non-smooth NMF [Pascual-Montano *et al.*, 2006] which uses an extra matrix factor that introduces an extra parameter, and change the way the target matrix is approximated.

The NMF models are defined as S4 classes that extends class `NMF`. All the available models can be retrieved calling the `nmfModel()` function with no argument:

```

nmfModel()
<Object of class: NMFstd >
features: 0
basis/rank: 0
samples: 0

```

One can specify the NMF model to use with a custom algorithm, using argument `model`. Here we first adapt a bit the custom algorithm, to justify and illustrate the use of a different model.

We use model `NMFOffset` [Badea L., 2008], that includes an offset to take into account genes that have constant expression levels accross the samples:

```
my.algorithm.offset <- function(x, seed, scale.factor=1){
  # do something with starting point
  # ...
  # for example:
  # 1. compute principal components
  pca <- prcomp(t(x), retx=TRUE)

  # retrieve the model being estimated
  data.model <- fit(seed)

  # 2. use the absolute values of the first PCs for the metagenes
  # Note: the factorization rank is stored in object 'start'
  factorization.rank <- nbasis(data.model)
  metagenes(data.model) <- abs(pca$rotation[,1:factorization.rank])
  # use the rotated matrix to get the mixture coefficient
  # use a scaling factor (just to illustrate the use of extra parameters)
  metaprofiles(data.model) <- t(abs(pca$x[,1:factorization.rank])) / scale.factor

  # 3. Compute the offset as the mean expression
  data.model@offset <- rowMeans(x)

  # return updated data
  fit(seed) <- data.model
  seed
}
```

Then run the algorithm specifying it needs model `NMFOffset`:

```
# run custom algorithm with NMF model with offset
nmf(V, 3, my.algorithm.offset, model='NMFOffset', scale.factor=10)
<Object of class: NMFfit >
# Model:
  <Object of class: NMFOffset >
  features: 50
  basis/rank: 3
  samples: 20
  offset: [ 0.4302509 1.323056 1.532638 1.182324 0.7698712 ... ]
# Details:
  algorithm: NMF.algo.6072a8ff
  seed: random
  RNG: 403, 389, -963168695, -1903204175, 1248152975, 1172799541, -1763377217, ... [7d13c1cc60f9b6390bf7e]
  distance metric: 'euclidean'
  residuals: 348.0784
  parameters:
  $scale.factor
  [1] 10

Timing:
```

```

user  system elapsed
0.000  0.000  0.002

```

### 3.2 Custom seeding method

The user can also define custom seeding method as a function of the form:

```

# start: object of class NMF
# target: the target matrix
my.seeding.method <- function(model, target){

  # use only the largest columns for W
  w.cols <- apply(target, 2, function(x) sqrt(sum(x^2)))
  metagenes(model) <- target[,order(w.cols)[1:nbasis(model)]]

  # initialize H randomly
  metaprofiles(model) <- matrix(runif(nbasis(model)*ncol(target))
                                , nbasis(model), ncol(target))

  # return updated object
  return(model)
}

```

To use the new seeding method:

```

nmf(V, 3, 'snmf/r', seed=my.seeding.method)
<Object of class: NMFfit >
# Model:
  <Object of class: NMFstd >
features: 50
basis/rank: 3
samples: 20
# Details:
algorithm:  snmf/r
seed:  NMF.seed.52899d1b
RNG: 403, 25, -671852523, -62925613, 1285853938, 874494436, 1934696155, ... [e0cc5847b800083c2460eb9eaa2
distance metric:  'euclidean'
residuals:  150.9747
Iterations: 115
Timing:
  user  system elapsed
0.450   0.000   0.539

```

## 4 Advanced usage

### 4.1 Package specific options

The package specific options can be retrieved or changed using the `nmf.getOption` and `nmf.options` functions. These behave similarly as the `getOption` and `options` base functions:

```
#show default algorithm and seeding method
nmf.options('default.algorithm', 'default.seed')
# retrieve a single option
nmf.getOption('default.seed')
# All options
nmf.options()
```

Currently the following options are available:

Option	Default value	Description
<code>default.algorithm</code>	<code>brunet</code>	Default NMF algorithm used by the <code>nmf</code> function when argument <code>method</code> is missing. The value should be the key of one of the available NMF algorithms. See <code>?nmfAlgorithm</code> .
<code>track.interval</code>	<code>30</code>	Number of iterations between two points in the residual track. This option is relevant only when residual tracking is enabled. See <code>?nmf</code> .
<code>error.track</code>	<code>FALSE</code>	Toggle default residual tracking. When <code>TRUE</code> , the <code>nmf</code> function compute and store the residual track in the result – if not otherwise specified in argument <code>.options</code> . Note that tracking may significantly slow down the computations.
<code>default.seed</code>	<code>random</code>	Default seeding method used by the <code>nmf</code> function when argument <code>seed</code> is missing. The value should be the key of one of the available seeding methods. See <code>?nmfSeed</code> .
<code>parallel.backend</code>	<code>mc</code>	Default parallel backend used by the <code>nmf</code> function when argument <code>.pbackend</code> is missing. Currently the following values are supported: <code>'mc'</code> for multicore, <code>'seq'</code> for sequential, <code>'sapply'</code> for <code>sapply</code> .
<code>verbose</code>	<code>FALSE</code>	Toggle verbosity.
<code>debug</code>	<code>FALSE</code>	Toggle debug mode, which is an extended verbose mode.

Table 4:

## 5 Session Info

- R version 2.14.0 (2011-10-31), x86\_64-unknown-linux-gnu
- Locale: LC\_CTYPE=en\_ZA.UTF-8, LC\_NUMERIC=C, LC\_TIME=en\_ZA.UTF-8, LC\_COLLATE=C, LC\_MONETARY=en\_ZA.UTF-8, LC\_MESSAGES=en\_US.UTF-8, LC\_PAPER=C, LC\_NAME=C, LC\_ADDRESS=C, LC\_TELEPHONE=C, LC\_MEASUREMENT=en\_ZA.UTF-8, LC\_IDENTIFICATION=C
- Base packages: base, datasets, grDevices, graphics, grid, methods, parallel, stats, tools, utils



- Other packages: Biobase~2.12.1, NMF~0.6.05, RColorBrewer~1.0-5, bigmemory~4.2.11, codetools~0.2-8, colorspace~1.1-0, digest~0.5.1, doMC~1.2.3, foreach~1.3.2, gridBase~0.4-3, iterators~1.0.5, multicore~0.1-7, stringr~0.5, synchronicity~1.0.13, xtable~1.6-0
- Loaded via a namespace (and not attached): compiler~2.14.0, plyr~1.6

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